SCHRIFTELIJK TENTAMEN

COSMOLOGY

1^{st} quarter 2023/2024

October 30, 2023

NOTE: THIS EXAM CONTAINS

- 4 short questions (1-4), page 2 to 3
- 1 major question (5), page 4 to 5

Grading:

- 4 short questions: each 1.25 point
- 1 major questions: each 4.0 point

Total grade is 1.0+ sum of these question points.

Please assure you have read all pages and questions.
Mention your name and studentnr. on ALL pages that you hand in.
Note: all equations that you may need are provided in the exam, and thus you are NOT allowed to consult lectures notes and book during the exam.

Question 1.:
the Big Bang: evidence (1.25 pt.)

List and discuss at least four (4) major arguments, ie. observational evidence, for the reality of the Hot Big Bang, ie. for the fact that the Universe evolved from a hot and dense state. Note: just listing is not sufficient, also argue why these observations/arguments are evidence for the Big Bang (this should be a brief justification).

Question 2.:
Luminosity Distance vs. Angular Diameter Distance (1.25 pt.)

Define the concepts of luminosity $D_L(z)$ and angular diameter distance $D_A(z)$ (note: specify definition with the help of an expression). How does their redshift dependence relate to the cosmic distance measure D,

$$D = R_c \, S_k \left(\frac{r}{R_c} \right) \tag{1}$$

in which $S_k(x)$ the curvature function,

$$S_{k=+1}(x) = \sin(x) \qquad S_{k=+0}(x) = x\,; \qquad S_{k=-1}(x) = \sinh(x),\,. \tag{2}$$

Explain the physical reason for the difference between these two fundamental distances (ie. the reciprocity relation between these 2 cosmic distances).

Question 3.:
Redshift (1.25 pt.)

Give the definition of redshift, in terms of wavelength of emitted and observed radiation. Subsequently, derive the relationship between the *cosmological redshift*, ie. redshift resulting from the expansion of the Universe, and the expansion factor $a(t)$ of the Universe.

Imagine a source has a cosmological redshift z_c. The source is moving with respect to the expanding Universe, causing a Doppler shift z_d. A telescope is looking at the core of the source (galaxy), and receiving light from near a black hole, with gravitational redshift z_{grav}. What will be the observed redshift z_{tot} of the received radiation (derive the answer) ?

Question 4.: General Relativity vs. Newton's Gravity (1.25 pt.)

Name at least 4 major differences between gravity described by General Relativity, and that described by Newton's gravity law (note: you only have to list an item, not to further justify/explain it).

General Relativity ultimately yields the Friedman-Robertson-Walker-Lemaitre equations,

$$\begin{aligned} \ddot{a} &= -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) a + \frac{\Lambda}{3} a \\ \dot{a}^2 &= \frac{8\pi G}{3} \rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3} a^2 \end{aligned} \qquad (3)$$

Which 3 aspects/physical influences are to be found in the FRWL equations that would never appear in Newton's gravity.

Question 5.: CMB and Recombination (4 pts.)

In this question we are going to investigate the origin and physics of the Cosmic Microwave Background, the radiation field of completely thermalized photons that fills the Universe.

a) During the formation of the CMB three processes occur very closely after each other. Describe these processes and their chronological order.

We can get a reasonable insight into the process in which the Universe evolves from a plasma of radiation, electrons and protons into a neutral medium of hydrogen atoms and freely propragating CMB photons. To this end we assume that the following reaction is an equilibrium reaction

$$p + e^- \rightleftharpoons H + \gamma\,, \tag{4}$$

where γ is a photon with the hydrogen ionization energy $E_{\rm ion} = 13.6\,eV$, which is the difference in energy between the free particles and the hydrogen atom,

$$E_{\rm ion} = (m_p + m_e - m_H)c^2\,. \tag{5}$$

As they are in equilibrium, the number density of each of the species (protons, electrons, Hydrogen atoms) is given by the Maxwell-Boltzmann eqn.

$$n_i = g_i \left(\frac{m_i kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_i c^2}{kT}\right)\,, \tag{6}$$

where m_i is the mass of these particles, T the temperature and k the Boltzmann constant. Note that statistical weights for hydrogen atoms, electrons and protons are $g_H = 4$, $g_p = 2$ and $g_e = 2$.

b) On the basis of the above, derive the Saha equation, i.e. the relation $n_H/(n_p n_e)$, where you may assume that $m_H \approx m_p$.

c) Given that the ionization fraction X of the Universe is defined as

$$X = \frac{n_p}{n_p + n_H} = \frac{n_e}{n_b}\,, \tag{7}$$

infer the expression for ionization fraction X as function of temperature T.

d) However, due to huge surplus of photons in our Universe recombination is delayed for a very long time after the temperature reaches the hydrogen ionization thresheld at $E_{\rm ion} = 13.6\,eV$. Explain why the Universe becomes neutral at a temperature of $T \approx 3000$K instead of the temperature suggested by the Saha equation. Which atomic transition of the hydrogen atoms is finally taking care of this (as opposed to the normal Lyman α transition).

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e) When and by what process were the CMB photons created?

f) Which three processes were responsible for transforming the spectral energy distribution of these photons into a blackbody spectrum? Why was Thomson scattering *not* sufficient for creating the blackbody spectrum?

g) The CMB is an (almost) perfect blackbody. What is the important, immediate consequence for cosmology?

SUCCES !!!!
BEDANKT VOOR JULLIE AANDACHT EN INTERESSE !!!!

Rien, Robbert, Fabian & Sonia